FACTORIZATION
AND
MULTIPLIERS OF SEGAL ALGEBRAS

by

Ronald Larsen

Wesleyan University, Middletown, Conn.
and
University of Oslo, Oslo

0. INTRODUCTION. Let G be a locally compact Abelian topological group, let $L_1(G)$ be the usual convolution group algebra of G, and let $C_o(G)$ be the algebra of continuous complex-valued functions on G that vanish at infinity. It is a well known factorization result that every element of $C_o(G)$ can be factored as the convolution product of an element in $L_1(G)$ and an element of $C_o(G)$, that is, $C_o(G) = L_1(G) * C_o(G)$ [13,p.283]. In this note we shall consider the question of whether or not $C_o(G) = S * C_o(G)$ for a given Segal algebra S in $L_1(G)$. The main result in this connection asserts that if the multiplier algebra of S properly contains $M(G)$, the space of bounded regular complex-valued Borel measures on G, then $S * C_o(G) \neq C_o(G)$. This theorem together with the machinery developed to prove it will also be applied to obtain some old and some new results about multipliers and factorization.

1. L_1-MODULES AND SEGAL ALGEBRAS. Before we turn to the proof of the theorem alluded to in the introduction, it seems appropriate

to recall some definitions and results that will be needed in the sequel. A Banach space $(V, \|\cdot\|_V)$ is said to be an L_1-module if there exists a multiplication operation between elements of $L_1(G)$ and elements of V, denoted by $\circ$, such that V is an algebraic module over $L_1(G)$ with respect to this multiplication and for which there exists some constant $B_V > 0$ such that $\|f\circ g\|_V \leq B_V\|f\|_1 \|g\|_V$ for every $f \in L_1(G)$ and $g \in V$. The symbol $\|\cdot\|_1$ denotes the usual norm in $L_1(G)$. If V is an L_1-module, then so is the dual space V^* of V provided we define the module product of $f \in L_1(G)$ and $x^* \in V^*$ by $(f\circ x^*)(g) = x^*(f\circ g)$, $g \in V$. In most of the specific cases to be considered below the module multiplication will be ordinary convolution with elements of $L_1(G)$ and will then be denoted, as usual, by $*$.

A Banach subalgebra $(S, \|\cdot\|_S)$ of $L_1(G)$ is said to be a Segal algebra if S is a translation invariant L_1-dense subalgebra of $L_1(G)$ such that for every $g \in S$ the mapping $s \to \tau_s g$ of G to S is continuous and $\|\tau_s g\|_S = \|g\|_S$, $s \in G$. The symbol $\tau_s g$ denotes the translate of g by s, that is, $\tau_s g(t) = g(t-s)$, $t \in G$. It follows from the conditions of the definition that a Segal algebra S is an ideal in $L_1(G)$, that there exists some constant $C > 0$ such that $\|g\|_1 \leq C\|g\|_S$, $g \in S$, and that $\|f*g\|_S \leq \|f\|_1\|g\|_S$, $f \in L_1(G)$ and $g \in S$. Without loss of generality we may and do assume that $C = 1$. In particular, it is evident that every Segal algebra is an L_1-module with convolution as the module multiplication. Every Segal algebra S contains an approximate identity that is bounded in L_1-norm. Such an approximate identity is bounded in the norm of S if and only if $S = L_1(G)$. If G is discrete, then there are no proper Segal algebras in $L_1(G)$. A discussion of these results concerning Segal algebras can be found in [21,pp.16-26,34-38].

Subsequently we shall wish to discuss some specific Segal algebras. We define them now. These and other examples are available in [20,pp.12, 127 and 131, 21,pp.23-26, 25].

(a) Let G be an infinite compact Abelian topological group. Then $C(G)$, the space of continuous complex-valued functions on G, with the supremum norm $\|\cdot\|_\infty$, and the usual L_p-spaces, $L_p(G)$, $1 < p < \infty$, are proper Segal algebras.

(b) Let G be a nondiscrete locally compact Abelian topological group, let $\hat{G}$ denote the dual group of G, and let $\hat{f}$ denote the Fourier transform of $f \in L_1(G)$. Then for each $p, 1 \leq p < \infty$,

$$A_p(G) = \{f \mid f \in L_1(G), \hat{f} \in L_p(\hat{G})\}$$

is a proper Segal algebra with the norm

$$\|f\|_{A_p} = \|f\|_1 + \|\hat{f}\|_p .$$

(c) Let $G = \mathbb{R}$, the real numbers, and denote by $M_1(\mathbb{R})$ the linear space of all continuous complex-valued functions f on $\mathbb{R}$ such that

$$\|f\|_{M_1} = \sup_{t \in \mathbb{R}} [\sum_{n=-\infty}^{\infty} (\max_{0 \leq s \leq 2\pi} |f(t+s+2\pi n)|)] < \infty .$$

Then $(M_1(\mathbb{R}), \|\cdot\|_{M_1})$ is a proper Segal algebra.

(d) Let $G = \mathbb{R}$ and let $L^A(\mathbb{R})$ denote the linear space of all those $f \in L_1(\mathbb{R})$ that are absolutely continuous and such that $f' \in L_1(\mathbb{R})$. Then $(L^A(\mathbb{R}), \|\cdot\|_{L^A})$ is a proper Segal algebra provided

$$\|f\|_{L^A} = \|f\|_1 + \|f'\|_1 .$$

If V and W are L_1-modules, then we denote by $V \otimes_\gamma W$ the <u>projective tensor product</u> of V and W. Every element $x \in V \otimes_\gamma W$ can be written in the form $x = \Sigma_{k=1}^{\infty} g_k \otimes h_k$ where $\{g_k\} \subset V$, $\{h_k\} \subset W$, and $\Sigma_{k=1}^{\infty} \|g_k\|_V \|h_k\|_W < \infty$. The projective tensor product $V \otimes_\gamma W$ is a Banach space with the greatest cross norm

$$\|x\|_\gamma = \inf\{\Sigma_{k=1}^{\infty} \|g_k\|_V \|h_k\|_W \mid x = \Sigma_{k=1}^{\infty} g_k \otimes h_k\} .$$

In particular, $\|g \otimes h\|_\gamma = \|g\|_V \|h\|_W$ for every $g \in V$ and $h \in W$. If K is the closed linear subspace of $V \otimes_\gamma W$ spanned by the elements of the form $(f \circ g) \otimes h - g \otimes (f \circ h)$, $f \in L_1(G)$, $g \in V$, and $h \in W$, then the quotient Banach space $V \otimes_\gamma W/K$ will be denoted by $V \otimes_{L_1} W$. This quotient space is called the L_1-<u>module tensor product</u> of V and W.

We denote by $\mathrm{Hom}_{L_1}(V,W)$ the Banach space of continuous linear transformations T from V to W such that $T(f \circ g) = f \circ Tg$, $f \in L_1(G)$ and $g \in V$. Such transformations are called <u>multipliers</u> or <u>module homomorphisms</u>. Tt is known that $\mathrm{Hom}_{L_1}(V,W^*)$ is isometrically isomorphic to $(V \otimes_{L_1} W)^*$, the dual space of $V \otimes_{L_1} W$. If V and W are Segal algebras, then it is a simple exercise, using the fact that V is L_1-norm dense in $L_1(G)$, to show that $T \in \mathrm{Hom}_{L_1}(V,W)$ if and only if T is a continuous linear transformation from V to W such that $T(f*g) = f*Tg = Tf*g$, $f,g \in V$. Moreover, in this case, it can be shown [3,14] that if T maps V into W, then $T \in \mathrm{Hom}_{L_1}(V,W)$ if and only if T is a continuous linear transformation that commutes with translation if and only if there exists a unique bounded continuous function φ defined on $\hat{G}$ such that $(Tf)\hat{} = \hat{\varphi}\hat{f}$ for every $f \in V$. The space of measures $M(G)$ can always be considered as a subspace of $\mathrm{Hom}_{L_1}(V,V)$ when V is a Segal algebra. This follows at once upon noting that if $\mu \in M(G)$ and

$f \in V$, then the usual convolution product $\mu * f$ belongs to V and $\|\mu * f\|_V \leq \|\mu\| \, \|f\|_V$, [21,p.20].

The results mentioned concerning tensor products can be found in [1,pp.230-237, 22,pp.445-461, 23,pp.71-73].

In the following sections we shall use the symbol $\simeq$ to denote "topological isomorphism", whereas $\cong$ will stand for "isometric isomorphism". The end of a proof is indicated by # .

2. THE MAIN THEOREMS. The first result of this section gives a more or less concrete description of $S \otimes_{L_1} W$ in the case that S is a Segal algebra and W is a Banach space of functions or measures on G that is an L_1-module with respect to the usual convolution product $*$. We shall call such an L_1-module W an L_1-<u>convolution module</u>. Besides Segal algebras, two examples of L_1-convolution modules that we shall utilize in the next section are $C_0(G)$ and $L_\infty(G)$, the space of essentially bounded measurable functions on G.

If S is a Segal algebra and W is an L_1-convolution module, then it is apparent that $g * h \in W$ for every $g \in S \subset L_1(G)$ and $h \in W$. Consequently, we can meaningfully define the linear space $S \underline{\otimes} W$ consisting of all those $u \in W$ of the form $u = \Sigma_{k=1}^{\infty} g_k * h_k$, where $\{g_k\} \subset S$, $\{h_k\} \subset W$, and $\Sigma_{k=1}^{\infty} \|g_k\|_S \|h_k\|_W < \infty$. It follows immediately from Theorem 6 of [24] that $S \underline{\otimes} W$ is a Banach space with the norm

$$|||u||| = \inf\{\Sigma_{k=1}^{\infty} \|g_k\|_S \|h_k\|_W \mid u = \Sigma_{k=1}^{\infty} g_k * h_k\} .$$

Moreover, if $u = \Sigma_{k=1}^{\infty} g_k * h_k \in S \underline{\otimes} W$, then

$$\begin{aligned} \|u\|_W &\leq \Sigma_{k=1}^{\infty} \|g_k * h_k\|_W \\ &\leq B_W \Sigma_{k=1}^{\infty} \|g_k\|_1 \, \|h_k\|_W \\ &\leq B_W \Sigma_{k=1}^{\infty} \|g_k\|_S \, \|h_k\|_W < \infty , \end{aligned}$$

where we conclude that $\Sigma_{k=1}^{\infty} g_k * h_k$ converges absolutely to u in W and $\|u\|_W \leq B_W |||u|||$.

THEOREM 1. <u>Let</u> G <u>be a locally compact Abelian topological group. If</u> S <u>is a Segal algebra and</u> W <u>is an</u> L_1<u>-convolution module, then</u> $S \otimes_{L_1} W \cong S \underline{\otimes} W$.

PROOF. Consider the mapping θ from $S \otimes_{\gamma} W$ to $S \underline{\otimes} W$ determined by $\theta(g \otimes h) = g * h$, $g \in S$ and $h \in W$. It is easily seen that θ is a norm decreasing surjective linear transformation that can be lifted in the canonical fashion to a norm decreasing surjective linear transformation Θ of $S \otimes_{L_1} W$ to $S \underline{\otimes} W$. Moreover, since θ is L_1-balanced, that is, $\theta[(f*g) \otimes h] = \theta[g \otimes (f*h)]$, $f \in L_1(G)$, $g \in S$, and $h \in W$, the series expansions of elements in $S \underline{\otimes} W$ converge absolutely in W, and S contains an approximate identity that is bounded in L_1-norm, we can use the same argument <u>mutatis mutandis</u> as employed in the proof of Theorem 3.3 in [23] to show that Θ is injective. We leave the details to the reader.

To show that Θ is an isometry it suffices, in view of [24, Theorem 6], to prove that if $\{g_k\} \subset S$ and $\{h_k\} \subset W$ are sequences such that $\Sigma_{k=1}^{\infty} \|g_k\|_S \|h_k\|_W < \infty$ and $\Sigma_{k=1}^{\infty} g_k * h_k = 0$, then $\Sigma_{k=1}^{\infty} T h_k(g_k) = 0$ for every multiplier $T \in \mathrm{Hom}_{L_1}(W,S^*)$. We observe first that $\mathrm{Hom}_{L_1}(W,S^*) \cong \mathrm{Hom}_{L_1}(S,W^*)$. Indeed, if $T \in \mathrm{Hom}_{L_1}(W,S^*)$, then the multiplier $T' \in \mathrm{Hom}_{L_1}(S,W^*)$ corresponding to T is defined by the formula $T'g(h) = Th(g)$, $g \in S$ and $h \in W$. Furthermore, we recall that if β denotes the isometry between $\mathrm{Hom}_{L_1}(S,W^*)$ and $(S \otimes_{L_1} W)^*$, then

$$\beta(T')(\Sigma_{k=1}^{\infty} g_k \otimes h_k) = \Sigma_{k=1}^{\infty} T' g_k(h_k)$$

for every $T \in \mathrm{Hom}_{L_1}(W,S^*)$ [23,p.72]

Since Θ is bijective and continuous, we see that the adjoint mapping $\Theta^* : (S \underline{\otimes} W)^* \to (S \otimes_{L_1} W)^*$ is also bijective [18,pp.96, 227 and 278] and consequently

$$\begin{aligned} \Sigma_{k=1}^{\infty} Th_k(g_k) &= \Sigma_{k=1}^{\infty} T' g_k(h_k) \\ &= \beta(T')(\Sigma_{k=1}^{\infty} g_k \otimes h_k) \\ &= (\Theta^*)^{-1}[\beta(T')](\Sigma_{k=1}^{\infty} g_k * h_k) \\ &= 0 , \end{aligned}$$

for every $T \in \mathrm{Hom}_{L_1}(W,S^*)$.

Thus $S \otimes_{L_1} W$ is isometrically isomorphic to $S \underline{\otimes} W$. #

The introduction of L_1-convolution modules is necessitated by the fact that for arbitrary L_1-modules the mapping θ defined in the proof of Theorem 1 may not be L_1-balanced.

Other versions of Theorem 1 have been given, for example, in [3,14,23,24].

If S is a Segal algebra and W is an L_1-convolution module then we set $S * W = \{g * h \mid g \in S \text{ and } h \in W\}$.

THEOREM 2. *Let* G *be a locally compact Abelian topological group, let* S *be a Segal algebra, let* W *be an* L_1*-convolution module, and suppose that*

(i) $S * W \subset C_o(G)$.

(ii) *There exists some constant* $B > 0$ *such that* $\|g * h\|_\infty \leq B\|g\|_S\|h\|_W$ *for every* $g \in S$ *and* $h \in W$.

If $S * W = C_o(G)$, *then* $\mathrm{Hom}_{L_1}(S,W^*) \simeq M(G)$.

PROOF. From assumptions (i) and (ii) we see that if $u = \Sigma_{k=1}^{\infty} g_k * h_k \in S \underline{\otimes} W$, then

$$\begin{aligned}\|u\|_\infty &\leq \Sigma_{k=1}^{\infty}\|g_k * h_k\|_\infty \\ &\leq B\,\Sigma_{k=1}^{\infty}\|g_k\|_S\|h_k\|_W < \infty ,\end{aligned}$$

and so $S \underline{\otimes} W \subset C_o(G)$ and $\|u\|_\infty \leq B\,|||u|||$, $u \in S \underline{\otimes} W$. Thus $S \underline{\otimes} W = C_o(G)$ since $C_o(G) = S * W \subset S \underline{\otimes} W$. Consequently, appealing to the Open Mapping Theorem [18,p.187], we see that the identity mapping from $S \underline{\otimes} W$ to $C_o(G)$ is a topological isomorphism, whence, by Theorem 1,

$$\begin{aligned}\mathrm{Hom}_{L_1}(S,W^*) &\cong (S \otimes_{L_1} W)^* \\ &\cong (S \underline{\otimes} W)^* \\ &\simeq (C_o(G))^* \\ &\cong M(G) . \quad \#\end{aligned}$$

Clearly, if S is a Segal algebra, then $S * C_o(G) \subset C_o(G)$ and $\|g * h\|_\infty \leq \|g\|_1\|h\|_\infty \leq \|g\|_S\|h\|_\infty$ for every $g \in S$ and $h \in C_o(G)$. Thus assumptions (i) and (ii) of Theorem 2 are fulfilled in the case that $W = C_o(G)$. Furthermore, we recall that $M(G)$ can always be considered as a subset of $\mathrm{Hom}_{L_1}(S,S)$ for any Segal algebra S .

THEOREM 3. *Let* G *be a locally compact Abelian topological group and let* S *be a Segal algebra. If* $S * C_0(G) = C_0(G)$, *then* $\mathrm{Hom}_{L_1}(S,S) \simeq M(G)$.

PROOF. First we observe the easily established fact that $\mathrm{Hom}_{L_1}(S,S) \subset \mathrm{Hom}_{L_1}(S,L_1(G))$. Secondly, since the multipliers in $\mathrm{Hom}_{L_1}(S,L_1(G))$ commute with translation and since a measure $\mu \in M(G)$ is absolutely continuous if and only if the mapping $s \to \tau_s\mu$ of G into $M(G)$ is continuous [17,p.251], we see that $\mathrm{Hom}_{L_1}(S,L_1(G)) = \mathrm{Hom}_{L_1}(S,M(G))$. Consequently, by Theorem 2 and the remarks preceding the statement of Theorem 3, we conclude that

$$\begin{aligned} M(G) &\subset \mathrm{Hom}_{L_1}(S,S) \\ &\subset \mathrm{Hom}_{L_1}(S,L_1(G)) \\ &= \mathrm{Hom}_{L_1}(S,M(G)) \\ &= \mathrm{Hom}_{L_1}(S,C_0(G)^*) \\ &\simeq M(G) , \end{aligned}$$

because $S * C_0(G) = C_0(G)$.

Therefore $\mathrm{Hom}_{L_1}(S,S) \simeq M(G)$. #

COROLLARY 1. *Let* G *be a locally compact Abelian topological group and let* S *be a Segal algebra. If* $\mathrm{Hom}_{L_1}(S,S) \not\simeq M(G)$, *then* $S * C_0(G) \neq C_0(G)$.

The converse of Theorem 3 fails to be valid. For example, let G be an infinite compact Abelian topological group and let $S = C(G)$. Then $\mathrm{Hom}_{L_1}(C(G),C(G)) \cong M(G)$ [17,p.74] and

$C(G) * C(G) \neq C(G)$. The latter assertion follows at once from the fact that $L_p(G) * L_{p'}(G) \subsetneq C(G)$, $1 < p < \infty$, $1/p + 1/p' = 1$ [4,p.91]. We shall give a new proof of this fact in the next section. See also [13,p.357].

3. APPLICATIONS. We first apply the theorems of the preceding section to the problem of characterizing multipliers. Some of the results have been proved elsewhere, but we include them here to illustrate the use of the theorems in section two.

THEOREM 4. <u>Let</u> G <u>be a locally compact Abelian topological group</u>.

(i) <u>If</u> G <u>is nondiscrete and</u> $1 \leq p \leq q < \infty$, <u>then</u> $\mathrm{Hom}_{L_1}(A_p(G), A_q(G)) \simeq (A_p(G) \underline{\otimes} C_o(G))^*$.

(ii) <u>If</u> G <u>is nondiscrete</u>, $1 \leq p < \infty$, <u>and</u> $1 < q \leq \infty$, <u>then</u> $\mathrm{Hom}_{L_1}(A_p(G), L_q(G)) \cong (A_p(G) \underline{\otimes} L_{q'}(G))^*$, $1/q + 1/q' = 1$.

(iii) <u>If</u> G <u>is infinite and compact</u>, <u>and</u> $1 < p < \infty$, <u>then</u> $\mathrm{Hom}_{L_1}(L_p(G), L_1(G)) \cong (L_p(G) \underline{\otimes} C(G))^*$.

(iv) <u>If</u> G <u>is infinite and compact</u>, $1 < p \leq 2$, $1/p + 1/p' = 1$, <u>and</u> $p' \leq q < \infty$, <u>then</u> $\mathrm{Hom}_{L_1}(L_p(G), A_q(G)) \simeq (L_p(G) \otimes C(G))^*$.

(v) <u>If</u> G <u>is infinite and compact</u>, <u>then</u> $\mathrm{Hom}_{L_1}(L_1(G), L_\infty(G)^*) \simeq M(G)$.

(vi) <u>If</u> G <u>is arbitrary</u>, <u>then</u> $\mathrm{Hom}_{L_1}(L_1(G), L_1(G)) \simeq M(G)$.

PROOF. On noting that $A_p(G) \subset A_q(G)$, $p \leq q$, an easy argument, utilizing the equivalent descriptions of multipliers of pairs of Segal algebras mentioned in section one, shows that

$$\begin{aligned}\mathrm{Hom}_{L_1}(A_p(G),A_q(G)) &= \mathrm{Hom}_{L_1}(A_p(G),L_1(G))\\ &= \mathrm{Hom}_{L_1}(A_p(G),C_o(G)^*)\ ,\end{aligned}$$

whence, from Theorem 1, we conclude that

$$\mathrm{Hom}_{L_1}(A_p(G),A_q(G)) \simeq (A_p(G)\underline{\otimes}\, C_o(G))^*\ .$$

Parts (ii) and (iii) follow at once form Theorem 1, whereas part (iv) is a consequence of part (iii) after noting that $\mathrm{Hom}_{L_1}(L_p(G),A_{p'}(G)) = \mathrm{Hom}_{L_1}(L_p(G),L_1(G))$.

Theorem 2 and the fact that $L_1(G) * L_\infty(G) = C(G)$ [13,p.283] combine to prove part (v), and part (vi) follows from Theorem 3 and $L_1(G) * C_o(G) = C_o(G)$. #

An immediate corollary of Theorem 4 is the next result.

COROLLARY 2. _If_ G _is a locally compact Abelian topological group, then the following spaces of multipliers are topologically isomorphic to the dual space of a Banach space of continuous functions on_ G :

(i) $\mathrm{Hom}_{L_1}(A_p(G),A_q(G))$ _for_ $1 \le p \le q < \infty$ _and_ G _nondiscrete_.

(ii) $\mathrm{Hom}_{L_1}(L_p(G),L_1(G))$ _for_ $1 < p < \infty$ _and_ G _infinite and compact_.

(iii) $\mathrm{Hom}_{L_1}(L_p(G),A_q(G))$ _for_ $1 < p \le 2$, $p' \le q < \infty$, $1/p + 1/p' = 1$, _and_ G _infinite and compact_.

Theorems 4 (i), (v) and (vi) and Corollary 2 (i) have also been proved elsewhere [3,14,17,pp.2-5,204-216, 22,pp.461 and 462]. Further examples of the use of tensor products to describe various multiplier spaces are available in [8,9,10,14,15,16,17,pp.180-190,23,24].

Next we shall apply Theorem 2 and Corollary 1 to obtain some results concerning factorization.

THEOREM 5. (i) If G is an infinite compact Abelian topological group, then

(a) $A_p(G) * C(G) \neq C(G)$, $1 \leq p < \infty$.

(b) $L_p(G) * L_{p'}(G) \neq C(G)$, $1 < p < \infty$, $1/p + 1/p' = 1$.

(ii) $M_1(\mathbb{R}) * C_o(\mathbb{R}) \neq C_o(\mathbb{R})$.

(iii) $L^A(\mathbb{R}) * C_o(\mathbb{R}) \neq C_o(\mathbb{R})$.

PROOF. Portion (a) of part (i) follows at once from Corollary 1 on recalling that $\mathrm{Hom}_{L_1}(A_p(G), A_p(G)) \not\leq M(G)$ [17,pp.207-208], whereas portion (b) is a consequence of Theorem 2 and the fact that $\mathrm{Hom}_{L_1}(L_p(G), L_p(G)) \not\leq M(G)$, $1 < p < \infty$ [17,pp.85 and 86].

Part (ii) also follows from Theorem 2 on noting that $M_1(\mathbb{R}) \subset L_1(\mathbb{R}) \cap C_o(\mathbb{R})$ [25,p.234] and $\mathrm{Hom}_{L_1}(M_1(\mathbb{R}), M(\mathbb{R})) = \mathrm{Hom}_{L_1}(M_1(\mathbb{R}), L_1(\mathbb{R})) \not\subset M(\mathbb{R})$ [5,p.265]. Part (iii) is evident from part (ii) and the fact that $L^A(\mathbb{R}) \subset M_1(\mathbb{R})$ [25,p.234]. #

It is also immediately apparent from Theorem 5 (i) (b) that for infinite compact groups G we have $L_p(G) * C(G) \neq C(G)$, $1 < p < \infty$, and $C(G) * C(G) \neq C(G)$.

For other factorization results the interested reader is referred to [2,4,6 ,pp.117-122,135, 7,pp.225 and 226, 11,12,13,pp.268-274, 282-285,337-338,354-357, 19,25,26,27].

If S is a Segal algebra, then the density of S in $L_1(G)$ combined with the fact that $L_1(G) * C_o(G) = C_o(G)$ reveals that the supremum norm closure of $S * C_0(G)$ is equal to $C_0(G)$. We have not been able to determine whether there exists proper Segal algebras S such that $S * C_0(G) = C_0(G)$.

REFERENCES

[1] F.F. Bonsall and J. Duncan, Complete Normed Algebras, Springer, Berlin - Heidelberg - New York, 1973.

[2] J.T. Burnham, Nonfactorization in subsets of the measure algebra, Proc. Amer. Math. Soc. 35 (1972), 104-106.

[3] J.T. Burnham, Multipliers of A-Segal algebras on LCA groups, I (preprint).

[4] R.E. Edwards, Approximation by convolutions, Pacific J. Math. 15 (1965), 85-95.

[5] R.E. Edwards, Operators commuting with translations, Pacific J. Math. 16 (1966), 254-265.

[6] R.E. Edwards, Fourier Series: A Modern Introduction, I, Holt, Rinehart, and Winston, New York, 1967.

[7] R.E. Edwards, Fourier Series: A Modern Introduction, II, Holt, Rinehart, and Winston, New York, 1967.

[8] A. Figà-Talamanca, Translation invariant operators in L^p, Duke Math. J. 32 (1965), 495-502.

[9] A. Figà-Talamanca and G.I. Gaudry, Density and representation theorems for multipliers of type (p,q), J. Austral. Math. Soc. 7 (1967), 1-6.

[10] G.I. Gaudry, Quasimeasures and operators commuting with convolution, Pacific J. Math. 18 (1966), 461-476.

[11] G.I. Gaudry, Bad behavior and inclusion results for multipliers of type (p,q), Pacific J. Math. 35 (1970), 83-94.

[12] S. Gulick, T.S. Liu, and A.C.M. van Rooij, Group algebra modules, II, Canad. J. Math. 19 (1967), 151-173.

[13] E. Hewitt and K.A. Ross, Abstract Harmonic Analysis, II, Springer, Berlin-Heidelberg-New York, 1970.

[14] H. Krogstad, Multipliers on homogeneous Banach spaces on compact groups, Report 11-73, Institut Mittag-Leffler, Djursholm, 1973.

[15] H.-C. Lai, On the multipliers of $A^p(G)$ - algebras, Tôhoku Math. J. 23 (1971), 641-662.

[16] H.-C. Lai, A characterization of the multipliers of Banach algebras, Yokohama Math. J. 20 (1972), 45-50.

[17] R. Larsen, An Introduction to the Theory of Multipliers, Springer, Berlin-Heidelberg-New York, 1971.

[18] R. Larsen, Functional Analysis: An Introduction, Dekker, New York, 1973.

[19] J.C. Martin and L.Y.H. Yap, The algebra of functions with Fourier transforms in L^p, Proc. Amer. Math. Soc. 24 (1970, 217-219.

[20] H. Reiter, Classical Harmonic Analysis and Locally Compact Groups, Oxford University Press, Oxford, 1968.

[21] H. Reiter, L^1-Algebras and Segal Algebras, Springer, Berlin-Heidelberg-New York, 1971.

[22] M. Rieffel, Induced Banach representations of Banach algebras and locally compact groups, J. Functional Analysis 1 (1967), 443-491.

[23] M. Rieffel, Multipliers and tensor products of L^p spaces of locally compact groups, Studia Math. 33 (1969), 71-82.

[24] R. Rigelhof, Tensor products of locally convex modules and applications to the multiplier problem, Trans. Amer. Math. Soc. 164 (1972), 295-307.

[25] H.-C. Wang, Nonfactorization in group algebras, Studia Math. 42 (1972), 231-241.

[26] L.Y.H. Yap, On the impossibility of representing certain functions by convolution, Math. Scand. 26 (1970), 132-140.

[27] L.Y.H. Yap, Ideals in subalgebras of the group algebras, Studia Math. 35 (1970), 165-175.